BLACK N WHITE

Learn Today Lead Tomorrow

jag....

NAME	
ROLL NUMBER	
SEMESTER	2nd
COURSE CODE	DCA1201
COURSE NAME	COMPUTER ORIENTED NUMERICAL METHODS

SET - I

Q1.a) Show that

$$\boldsymbol{\delta\mu = \frac{1}{2}(\Delta + \nabla)}$$

Solution .:- Assuming
$\mathit{\Delta} and\ \nabla\ are\ linear\ operators\ acting\ on\ same\ function\ f(x),$

$we\ can\ write\ them\ as:$

$$\Delta f = Af$$

$$\nabla f = Bf$$

Where A and B are matrices of differential operators acting on f .

→Now we assume $\delta\mu\ can\ be\ expressed\ in\ terms\ of\ \ \mathit{\Delta} and\ \nabla\ \ as\ \delta\mu\ = Cf$

Where C is another matrix of differential operator . We aim to show that

$$c = \frac{1}{2}(A + B)$$

To demonstrate this, let's consider the action of $\delta\mu\ on\ f:$

$$\delta\mu f = \frac{1}{2}(\mathit{\Delta} f + \nabla f)$$

By substituting $\mathit{\Delta} f and\ \nabla f\ in\ terms\ of\ A\ and\ B\ we\ get:$

$$\delta\mu f = \frac{1}{2}(Af + Bf)$$

$$\delta\mu f = \frac{1}{2}(A + B)f$$

Since A and B are the matrices of differential operators corresponding to
$\mathit{\Delta} and\ \nabla\ respectively\ , we\ see\ that:$

$$c = \frac{1}{2}(A + B)$$

Thus :-
$\delta\mu f = \frac{1}{2}(\mathit{\Delta} + \nabla)\ proof$

Q1.b) $\Delta - \nabla = \Delta\nabla$

Solution .:- Let's $\textbf{\textit{denote the}}\ \Delta and\ \nabla\ on\ a\ function\ f\ as\ follows:$

- $\Delta f\ :\ Applying\ the\ \Delta\ operator\ to\ f\ .$
- $\nabla f: Applying\ the\ \nabla\ operator\ to\ f.$
- $\Delta(\nabla f): Applying\ the\ \Delta\ operator\ to\ the\ result\ of\ \nabla f\ .$
  $Given\ the\ relationship\ \ \Delta - \nabla$
  $= \Delta\nabla\ , we\ need\ to\ show\ that\ for\ any\ function\ f\ :$
  $(\Delta - \nabla)f = (\Delta\nabla)f$
  $Applying\ \ \Delta f - \nabla f = \Delta(\nabla f)$
  $\rightarrow\ Let's\ analyze\ the\ left - hand\ side$:

$\Delta f - \nabla f$

This is simply the difference between the application of $\Delta and\nabla\ on\ f$.

Now, let's consider the right-hand side:

$(\Delta\nabla)f$

This means first applying $\nabla to\ f, then\ applying\ \Delta\ to\ the\ result\ of\ \nabla f.$

To prove the equality, we need to check if this holds for an arbitrary function *f.*

We assume linearity and certain properties of the operators. Consider the scenario where $\Delta\ and\ \nabla$ are specific linear operators that follow this relationship by definition.

By substituting specific forms of $\Delta\ and\ \nabla$ (e.g., difference operators, specific matrices), you can derive this explicitly. For simplicity, let's consider an example with matrices:

Let $\Delta = A\ and\ \nabla = B\ \ , where\ A\ and\ B\ are\ matrices\ . We\ need\ to\ show\ :$

$$A - B = AB$$

This means we need *A* and *B* such that this equality holds for all vectors *f* .

Suppose *A* and *B* are specific matrices that satisfy this relationship. For example

If :-

$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Then

$AB = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$A - B = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In this specific case *A – B* dose not equal AB , but by carefully choosing *A and B ,* we can achieve the equality . For instance let's A = 1 and B = -1 :

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Then

$AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$A - B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

In this case , $A - B$ still does not equal AB . Therefore , for this equality to hold Δ *and* ∇ must have specific interdependent properties. The choice of such operators depends on the context and the properties of these operators.

This relationship suggests a deeper, possibly problem-specific structure. Therefore, while abstract, proving or finding a general proof depends on specific context or assumptions about Δ *and* ∇ .

Q.2) Find Lagrange's interpolation polynomial fitting the points $y(1) = -3$, $y(3) = 0$, $y(4) = 30$, $y(6) = 132$ Hence find y(5).

Solution.:-

Given the points$(x_0 , y_0) = (1,-3),(x_1,y_1) = (4,30)$, and $(x_3,y_3)=(6,132)$, we compute the Lagrange basis polynomials $L_i(x)$:

1. *$L_0(x)$ for $x_0 = 1$:*

   $L_0(x)=\frac{(x-3)(x-4)(x-6)}{(1-3)(1-4)(1-6)} = \frac{(x-3)(x-4)(x-6)}{(-2)(-3)(-5)} = \frac{(x-3)(x-4)(x-6)}{30}$

2. *$L_0(x)$ for $x_1 = 3$:*

   $L_1(x)=\frac{(x-1)(x-4)(x-6)}{(3-1)(3-4)(3-6)} = \frac{(x-3)(x-4)(x-6)}{(2)(-1)(-3)} = \frac{(x-1)(x-4)(x-6)}{6}$

3. *$L_2(x)$ for $x_2 = 4$:*

   $L_0(x)=\frac{(x-1)(x-3)(x-6)}{(4-1)(4-3)(4-6)} = \frac{(x-1)(x-3)(x-6)}{(3)(1)(-2)} = \frac{(x-1)(x-3)(x-6)}{6}$

4. *$L_3(x)$ for $x_3 = 6$:*

   $L_0(x)=\frac{(x-1)(x-3)(x-4)}{(6-1)(6-3)(6-4)} = \frac{(x-1)(x-3)(x-4)}{(5)(3)(2)} = \frac{(x-1)(x-3)(x-4)}{30}$

Now, we form the Lagrange interpolation polynomial P(x):

$P(x) = y_0L_0(x) + y_1L_1(x) + y_2L_2(x) + y_3L_2(x)+y_3L_3(x)$

Substituting the values $y_0= -3y_1 = 0$, $y_2 = 30$, and $y_3 = 132$, we get :

$$P(x) = -3\left(\frac{(x-3)(x-4)(x-6)}{30}\right) + 0\left(\frac{(x-1)(x-4)(x-6)}{6}\right)$$
$$+ 30\left(-\frac{(x-1)(x-3)(x-6)}{6}\right) + 132\left(\frac{(x-1)(x-3)(x-4)}{30}\right)$$

Simplifying each term separately:

$$-3\left(\frac{(x-3)(x-4)(x-6)}{30}\right) = -\frac{3}{30}(x-3)(x-4) = -\frac{(x-3)(x-4)(x-6)}{10}$$

$$Since\ y1 = 0\ , the\ term\ involving\ L1(x) is\ zero\ .$$

$30\left(-\frac{(x-1)(x-3)(x-6)}{6}\right)$ = -5(x-1)(x-3)(-6)

$132\left(\frac{(x-1)(x-3)(x-4)}{30}\right) = \frac{132}{30}(x-3)(x-4) = \frac{22}{5}(x-1)(x-3)(x-4)$

Combining these, we get:

$P(x) = -\frac{(x-3)(x-4)(x-6)}{10} - 5\,(x-1)(x-6) + \frac{22}{5}(x-1)(x-3)(x-4)$

To find y(5) , we evaluate P(5) :

$P(5) = -\frac{(5-3)(5-4)(5-6)}{10} - 5\,(5-1)(5-3)(5-6) + \frac{22}{5}(5-1)(5-3)(5-4)$

Calculating each term:

$-\frac{(5-3)(5-4)(5-6)}{10} = -\frac{(2)(1)(-1)}{1} = \frac{2}{10}$= 0. 2

$-5(5-1)(5-3)(5-6) = -5(4)(2)(-1) =$ -5 x 8 x (-1) = 40

$\frac{22}{5}(5-1)(5-4) = \frac{22}{5}(4)(2)(1) = \frac{22}{5}$ x8 = $\frac{176}{5} = 35.2$

Combining these:

P(5) = 0.2+40+35.2 = 75.4

Therefore , y(5) = 75.4

Q3) Evaluate $f(15)$, given the following table of values:

x	10	20	30	40	50
Y=f(x)	**46**	**66**	**81**	**93**	**101**

Solution .:- we need to estimate*(15).* One way to do this is by linear interpolation between the points (10,46) and(20,66). \

First, let's determine the equation of the line passing through these two points.

The slope *m* of the line through (10,46) and(20,66) is given by :

$$m = \frac{f(20) - f(10)}{20 - 10} = \frac{66 - 46}{20 - 10} = \frac{20}{10} = 2$$

Using the point-slope form of the line equation $y - y1 = m(x - x_1)$ with the point (10,46) , we get :

$$y - 46 = 2(x-10)$$

Solving for y , we have :

$$y=2(x-10) +46$$

$$y=2x-20+46$$

$$y=2x+26$$

Now , we substitute x = 15 into this equation to find f(15):

f(15)=2(15)+26=30+26=56

thus ,

the estimated value of f(15) is

56 *ans*

SET - II

Q4) Find the equation of the best fitting straight line for the data:

x	1	3	4	6	8	9	11	14
Y	1	2	4	4	8	7	8	9

Solution .:-

First , we compute the necessary summation :

$$\sum x = 1 + 3 + 4 + 6 + 8 + 9 + 11 + 14 = 56$$

$$\sum y = 1 + 2 + 4 + 4 + 5 + 7 + 18 + 9 = 56$$

$$\begin{aligned}\sum x^2 &= 1^2 + 3^2 + 4^2 + 6^2 + 8^2 + 9^2 + 11^2 + 14^2 \\ &= 1 + 9 + 16 + 36 + 64 + 81 + 121 + + 196 \\ &= 524 + 2^2 + 4^2 + 4^2 + 5^2 + 7^2 + 18^2 + 9^2 \\ &= 1 + 4 + 16 + 16 + 25 + 49 + 64 + 81 \\ &= 256 + 1 + 3.2 + 4.4 + 6.4 + 8.5 + 9.7 + 11.8 + 14.9 + 6 \\ &+ 16 + 24 + 40 + 63 + 88 + 126 = 364\end{aligned}$$

Now we substitute these values into formulas for *m* and *b* :

$$m = \frac{n \sum xy - (\sum x)\,(\sum y)}{n\,\Sigma^2 - (\sum x)^2}$$

$$m = \frac{8.364 - 56.40}{8.524 - 56^2} = \frac{2912 - 2240}{3136 - 3136} = \frac{672}{1056} = \frac{2}{3}$$

$$b = \frac{\sum y - m \sum x}{n}$$

$$m = \frac{40 - \frac{2}{3}.56}{8} = \frac{40 - \frac{112}{3}}{8} = \frac{8}{24} = \frac{1}{3}$$

Therefore , the equation on the best fitting straight line is :

$$y = \frac{2}{3}x + \frac{1}{3}.$$

So , the equation the line of best fit is $y = \frac{2}{3}x + \frac{1}{3}$.

Q.5 For what value of λ & μ the following system of equations:

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$ **may have**

(i) Unique solution

(ii) Infinite number of solutions

(iii) No solution

Solution :

We will use matrix methods to analyze the system. First, we write the augmented matrix for the system:

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 1 & 2 & 3 & | & 10 \\ 1 & 2 & \lambda & | & \mu \end{bmatrix}$$

We perform row operations to bring the matrix to row echelon form.

1. Subtract row 1 from rows 2 and 3:

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 2 & | & 10 \\ 0 & 0 & \lambda - 3 & | & \mu - 4 \end{bmatrix}$$

2. Subtract row 2 from row 3:

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 2 & | & 4 \\ 0 & 0 & \lambda - 3 & | & \mu - 10 \end{bmatrix}$$

Now , we analyze the last row $0.x + 0.y + (\lambda - 3)z = \mu - 10$:

i) **Unique Solution**

For the system to have a unique solution, the coefficient of z in the third equation must be non-zero :

$$\lambda - 3 \neq 0 \; or \; \lambda \neq 3$$

If $\lambda \neq 3$, the system has a unique solution .

ii) **Infinite Number of Solutions**

For the system to have an infinite number of solutions, the third equation must be a multiple of the previous equations, which means the last row must be:

$$0 = 0$$

This happens when : $\lambda - 3 = 0 \; and \; \mu - 10 = 0$

So $\lambda = 3 \;\; and \;\; \mu = 10$

iii) **No Solution**

For the system to have no solution, the third equation must contradict the previous equations, which means the last row must be:

0 = non-zero constant

This happens when : $\lambda = 3 \;\; and \;\; \mu \neq 10$

So $\lambda = 3 \;\; and \;\; \mu \neq 10$

- **Unique Solution : $\boldsymbol{\lambda \neq 3}$**
- **Infinite Number of Solutions : $\boldsymbol{\lambda = 3 \;\; and \;\; \mu = 10}$**
- **No Solution: $\boldsymbol{\lambda = 3 \;\; and \;\; \mu \neq 10}$**

Q.6 Find the solution for $\boldsymbol{x = 0.2}$ taking interval length 0.1 using Euler's method to solve: $\boldsymbol{\frac{dy}{dx} = 1 - y}$ given $\boldsymbol{y(0) = 0}$.

Solution :

Steps for Euler's Method

1. **Initial Condition** $x_o = , y_o = 0$
2. **Step size** $h = 0.1$
3. **Iterative Formula** $y_x+1=y_n+h.f(x_n,y_n)$

Iterations

Iterations 1st

- $x_0 = 0$
- $y_0 = 0$
- $f(x_0 , y_0) = 1- y_0 = 1 – 0 = 1$

Using the iterative formula : $y_1=y_0+h.f((x_0,y_0)=0+0.1.1=0.1$

Update *x* **:** $x_1=x_0+h=0+0.1=0.1$

So , after the first iteration : $x_1=0.1 , y_1 = 0.1$

Iterations 2nd

- $x_1 = 0.1$
- $y_1 = 0.$
- $F(x_1,y_1)=1-y_1=1-0.1=0.9$

Using the iterative formula :

$$y2=y1+h.f(x1,y1) = 0.1+0.1.0.9=0.1+0.09=0.19$$

Update *x :* $x_2=x_1+h=0.1+0.1=0.2$

So, after the second iteration: $x_2 = 0.2$, $y_2 = 0.19$

⇨ *Using Euler's method with a step size of h = 0.1 , we find the approximate solution for y at x = 0.2 to be :*

$$\boldsymbol{y(0.2) \approx 0.19}$$

Therefore, the solution using Euler's method to solve $\boldsymbol{\frac{dy}{dx} = 1 - y \text{ with } y(0) = 0 \text{ at } x = 0.2 \text{ is } y \approx 0.19}$